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Benchmarking New Adaptive Equalizers Leveraging a Different Robust Adaptive Algorithms: NLMS, APA, PAP, and MRNQ

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Abstract- Context: Numerous adaptive equalization techniques have been developed in the literature to effectively mitigate inter-symbol interference (ISI) in digital communication systems. **Objective:** This study presents a comparative analysis of four adaptive algorithms (NLMS, APA, PAP, and MRNQ) implemented within a decision feed-forward equalizer (DFE) structure. The objective is to identify the strengths and limitations of each algorithm under identical transmission conditions and to provide guidance for selecting appropriate equalization strategies for practical applications in modern communication systems. **Method:** The performance of each equalizer is evaluated using multiple criteria, including computational complexity, constellation diagram analysis, compliance with the Nyquist criterion, and mean squared error (MSE). **Results:** The results show that all four algorithms effectively reduce ISI and achieve satisfactory signal equalization. In particular, APA-DFE demonstrates faster convergence, higher accuracy, DFE-PAP provides a favorable trade-off between performance, and complexity, MRNQ-DFE exhibits a high convergence speed, while NLMS-DFE shows the weakest performance among the considered methods. **Conclusions:** The study identifies DFE-PAP as the most efficient and practical solution, offering robust ISI mitigation with relatively low computational cost. Although all algorithms perform adequately, DFE-PAP clearly outperforms the others in terms of convergence speed and accuracy, making it well suited for real-world implementations

Keywords- Digital communication system, Decision feed forward equalizer, Convergence speed, Computational complexity, NLMS algorithm, APA, PAP algorithm, MRNQ algorithm.

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1. Introduction

In modern digital communication systems, reliable data transmission relies heavily on effective equalization techniques [1, 2], especially in challenging environments affected by multipath propagation and fading. In wireless channels, transmitted signals often arrive at the receiver via multiple paths with different delays, leading to inter-symbol interference (ISI) that severely impairs performance. Adaptive equalizers are designed to address this issue by continuously adjusting their parameters in real time, thereby improving signal quality and system robustness.

At the heart of these equalizers lie adaptive algorithms, which are essential for handling time-varying channel conditions. Their main function is to automatically update the filter coefficients of the equalizer using real-time input data and error feedback, without requiring prior knowledge of the channel. This capability enables the equalizer to track channel variations and maintain optimal performance, making adaptive algorithms highly suitable for applications such as mobile communications, broadband systems, and wireless data transmission.

In particular, the integration of adaptive algorithms into structures such as the decision feed-forward equalizer (DFE) offers significant improvements in ISI suppression and signal recovery. The effectiveness of such equalizer structures depends largely on the choice of the adaptive update algorithm, which influences convergence speed, steady-state error, and robustness to channel dynamics. To assess performance under various conditions, this study investigates the DFE structure combined with four different adaptive algorithms that we have recently proposed [3,4,5], each presenting distinct trade-offs in terms of computational complexity, stability, and convergence speed.

This paper is organized as follows: Section II presents the digital communication system considered in this study. Section III introduces the decision feed-forward equalizer (DFE) structure, followed by Section IV, which details the adaptive algorithms integrated with the DFE. In Section V, we analyze the computational complexity of each algorithm. Section VI provides a performance comparison among the four equalizers. Finally, Section VII concludes the paper.

2. Digital communication system

The digital communication system model that we consider in this paper is described in Figure 1.

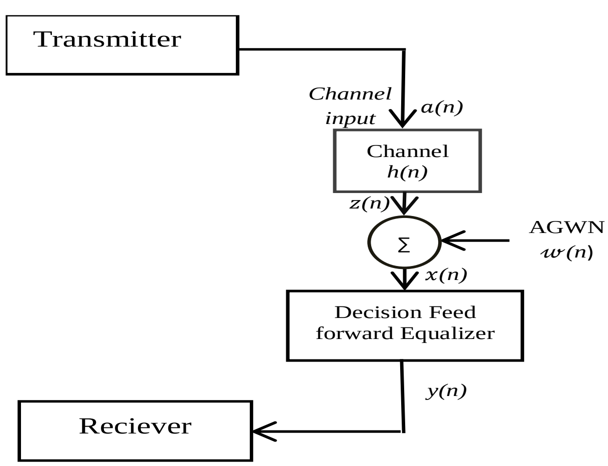

Figure 1. Digital communication channel scheme used.

In this configuration, the transmitted signal propagates through a multipath fading channel before reaching the receiver. Due to reflections, scattering, and other propagation phenomena, the channel introduces multiple delayed and attenuated copies of the original signal. These overlapping signal components interfere with one other, causing inter-symbol interference (ISI), which poses a significant challenge to accurate signal detection and overall system performance.

The output of the channel is denoted by $x(n)$, and is expressed as:

$$x(n) = z(n) + w(n) \tag{1}$$

$$x(n) = a_0(n) * h_0(n) + \sum_{m=1}^{M} a_m(n) * h_m(n) + w(n) \tag{2}$$

In this formulation, $a_0(n) * h_0(n)$ represents the convolution between the reference signal and the impulse response of the main transmission channel. The second term, $\sum_{m=1}^{M} a_m(n) * h_m(n)$, corresponds to the cumulative interference caused by M co-channel signals, where m=1,2,…M, , and M denotes the total number of interfering sources. The term $w(n)$ refers to additive white Gaussian noise (AWGN).

To model the multipath fading characteristics of the wireless channel, a Rayleigh fading model is employed. This model is based on the statistical behavior of the channel amplitude |h|, which follows the probability density function:

$$p(h) = \frac{h}{\sigma^2} e^{-\frac{h^2}{2\sigma^2}} \tag{3}$$

Here, the channel coefficient $h = h_{re} + jh_{im}$, where both the real and imaginary parts are independent, identically distributed (i.i.d.) Gaussian random variables with zero mean and variance $\sigma^2 = 1$.

The Monte Carlo method is used for simulation purposes, with the number of samples set to 100 to ensure statistical reliability of the results.

3. Decision feed-forward equalizer structure

In this study, we employ a decision feed-forward equalizer (DFE), as illustrated in Fig. 2, to mitigate inter-symbol interference (ISI) introduced by the dispersive communication channel. The equalizer relies solely on a feed-forward structure and consists of an adaptive finite impulse response (FIR) filter followed by a decision device. Its objective is to compensate for channel-induced amplitude and phase distortions and to recover the transmitted symbol sequence.

Let $a(n)$ denote the transmitted symbol sequence, $x(n)$ the received signal at the channel output, and $w(n) = [w_0(n), w_1(n), \ldots., w_{L-1}(n)]^T$ the coefficient vector of an L-tap FIR feed-forward filter. The equalizer output is given by:

$$y(n) = w^T(n)x(n) \tag{4}$$

where $x(n) = [x(n), x(n-1), \ldots, x(n-L+1)]^T$is the input vector.

A decision device (quantizer) $Q[\cdot]$ maps the equalizer output to the nearest constellation point, producing the detected symbol:

$$\tilde{a}(n–\Delta) = Q[y(n)] \tag{5}$$

where Δ represents the decision delay introduced to align the equalizer output with the desired symbol. Under ideal equalization conditions, this estimate satisfies

$$\tilde{a}(n-\Delta) \approx a(n-\Delta) \tag{6}$$

indicating that ISI has been effectively suppressed.

The adaptive process operates in two distinct modes:

Training mode:

At the beginning of transmission, a known training sequence $a(n)$ is sent. The receiver has access to this sequence and uses it as a reference to adjust the equalizer coefficients. The error signal is computed as:

$$e(n) = a(n-\Delta) - y(n) \tag{7}$$

which drives the adaptation algorithm to minimize the mean square error (MSE), thereby forcing the equalizer output to match the known symbols.

Decision mode:

After convergence, the system switches to decision mode, where no reference symbols are available. The detected symbols at the equalizer output are fed back as a surrogate reference:

$$e(n) = \tilde{a}(n-\Delta) - y(n) \tag{8}$$

In this mode, the equalizer continues to track slow channel variations using its own decisions, assuming that the probability of decision errors remains low.

In both modes, the filter coefficients $w(n)$ are updated by minimizing the MSE criterion using the chosen adaptive algorithm. This two-stage operation enables rapid initial convergence during training and sustained performance during data transmission without the need for continuous pilot symbols.

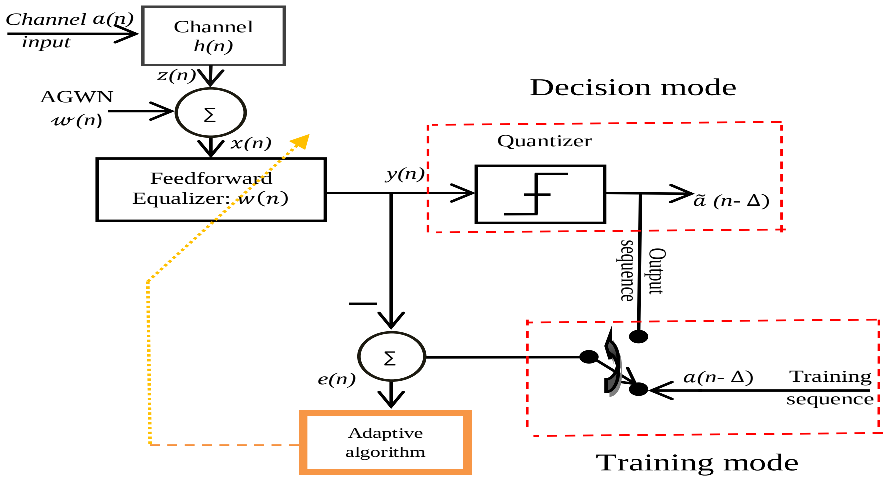

Figure 2. Structure of decision feed-forward equalizer.

4. Adaptive algorithms

It has been demonstrated in [1,3–5] that the DFE equalizer is effective in mitigating ISI in digital communication systems when combined with various adaptive algorithms. Each algorithm offers specific advantages and limitations in terms of convergence speed, computational complexity, and robustness. In the following, we present the algorithms employed in this study.

4.1 The NLMS Algorithm

The normalized least mean squares (NLMS) algorithm is combined with the decision feed-forward equalizer (DFE) structure to form the NLMS-DFE equalizer [3-5]. This equalizer iteratively updates the feed-forward filter coefficients $w(n)$ to minimize the mean squared error (MSE) between its output $\boldsymbol{y}(n)$ and the desired signal $\tilde{a}(n-\Delta)$, where Δ denotes a delay parameter.

The coefficient update equation for $w(n)$ is given in vector form by [3,4,6]:

$$w(n) = w(n-1) + \mu\, x(n)[x^T(n)x(n)]^{-1}e(n) \tag{9}$$

Here, $x(n) = [x(n), x(n-1), \dots, x(n-L+1)]^T$ is the input signal vector containing L samples for the feed-forward filter, while $w(n) = [w_0(n), w_1(n), \dots, w_{L-1}(n)]^T$ represents the adaptive filter coefficient vector. The superscript T denotes the transpose operator, which involves both transposition and complex conjugation of the vector elements.

The error signal $e(n)$, defined in (7 and 8), quantifies the difference between the filter output and the desired response at each iteration. The step-size parameter μ controls the adaptation rate and must be carefully chosen within the interval (0,1) to ensure stable and efficient convergence of the algorithm.

This NLMS-DFE scheme effectively combines the robustness of the normalized LMS algorithm with the interference mitigation capability of the DFE, resulting in an adaptive equalizer capable of compensating for inter-symbol interference in communication channels.

4.2 The Affine Projection Algorithm

The affine projection algorithm of order P, which exploits multiple input samples, is combined with a DFE structure to form the APA-DFE equalizer [3,5]. This approach provides satisfactory performance in the field of adaptive equalization.

In the APA-DFE, the adaptive filter $w(n)$ is adjusted according to the following update equations [3,5,7]:

$$w(n) = w(n-1) + \mu X(n)[X(n)^T X(n)]^{-1}e(n) \tag{10}$$

$$e(n) = a(n-\Delta) - X^T(n)w(n+1) \quad (training\ mode)$$

$$= \tilde{a}(n-\Delta) - X^T(n)w(n+1) \quad (decision\ mode) \tag{11}$$

here: $e(n)$ is the a priori error

The input matrix $X(n)$ is a $P \times L$ matrix, composed of the last P input vectors of the equalizer :

$$X(n) = [x(n), x(n-1), \dots, x(n-P+1)]^T \tag{12}$$

$x(n)$ is the input signal vector of L samples for the feed-forward filter, given by:

$$x(n) = [x(n) \cdots \cdots x(n - L + 1)]^T \tag{13}$$

where, T denotes the transpose operator, L represents the filter length, P is the projection order, and μ is the step-size parameter that should be chosen within the range $0 < \mu < 1$.

4.3 The Pseudo Affine Projection Algorithm

The combination of the PAP algorithm with the DFE structure results in the DFE-PAP equalizer [4]. The PAP algorithm benefits from the convergence of actively weighted values and, importantly, operates independently of the statistical properties of the input signal, which typically exhibits a discrete nature [8]. Unlike the conventional affine projection (AP) algorithm, the PAP method integrates the projection mechanism with an iterative update of the initial point, progressively moving toward the desired projection on a convex manifold. This process involves the inclusion of an additional pseudo-function in the weight update rule.

We initiate our algorithm by updating the pseudo-affine projection based on the filtering error and the feed-forward filter coefficients of the equalizer, as given by the following relations:

$$w(n) = w(n-1) + \mu\, X^{+}(n) e(n) \tag{14}$$

where :

The term $\mathrm{e}(n)$ represents the a priori error:

$$e(n) = \begin{cases} a(n-\Delta) - y(n) = a(n-\Delta) - X(n)w(n-1), \ \text{at traning mode} \\ \tilde{a}(n-\Delta) - y(n) = \tilde{a}(n-\Delta) - X(n)w(n-1), \ \text{at decision mode} \end{cases} \tag{15}$$

During implementation, we use a training period, during which the known symbols are used as $\tilde{a}(n-\Delta)$, $\tilde{a}(n-\Delta) = a(n-\Delta)$. After this period, the equalizer switches to operational mode, replacing the training sequence with decisions, i.e., $\tilde{a}(n\text{-}\Delta) = Q[y(n)]$, as explained in Section 3.

$\boldsymbol{y}(n)$ represents the output vector of the adaptive equalizer:

$$y(n) = [y(n) \ldots \ldots y(n - P + 1)]^T \tag{16}$$

$X^{+}(n)$ is a matrix representing the Moore–Penrose pseudo-inverse of the matrix $X^{\boldsymbol{T}}(n)$:

$$X^{+}(n) = X^T(n)\, [X(n)\, X^T(n)]^T \tag{17}$$

The input matrix $X(n) = [x(n), \ldots, x(n-P+1)]^T$ is a $P \times L$ matrix that contains the most recent P input vectors of the equalizer. L and P denote the segment length and the projection order, respectively. The parameter μ is a scalar that controls both the stability and the convergence rate, with a value chosen between 0 and 1.

$x(n)$ denotes the input signal vector consisting of L samples for the feed-forward filter as defined in (13).

The vector $u(n)$ corresponds to the optimal linear prediction error of the feed-forward filter equalizer, and is expressed as:

$$u(n) = [x(n)x(n-1)\dots x(n-P+1)]\acute{a}_P \quad (18)$$

Since the input signal of the equalizer is a stationary random signal, we represent the vector $\acute{a}_P$ as an optimal forward linear prediction coefficient vector of the feed-forward filter equalizer, $\acute{a}_P = [1\ \acute{a}_1 \quad \acute{a}_2 \dots \acute{a}_{P-1}]^T$, and is defined as:

$$\acute{a}_P = L^{-1}[\,X^T(n)X(n)]^{-1}[\,E_{P-1}\ 0 \dots 0]^T \quad (19)$$

E_{P-1} presents the prediction error energy which is related to the vector $u(n)$ through the following relation:

$$E_{P-1} = u^T(n)x(n) \quad (20)$$

The a posteriori error $\hat{\boldsymbol{e}}(n)$ is estimated using the following formula for the a posteriori error vector P:

$$\hat{e}(n) = e(n) - X(n)w(n) \quad (21)$$

By combining (14) and (21), we obtain the following relation between the adaptation step size and the a priori error:

$$\hat{e}(n) = (1-\mu)e(n) \quad (22)$$

In the following, we assume the adaptation step size in the optimal case $\mu = 1$. Under this assumption, relation (22) reduces to the zero vector [9], and the update equation of the feed-forward filter of the proposed equalizer can be rewritten as follows [10]:

$$w(n) = w(n-1) + [X^T(n)\,X(n) + \delta I]^{-1}X(n)e(n) \quad (23)$$

where δI is a regularization identity matrix.

By substituting relations (18) and (20) into relation (23), we obtain the feed-forward filter update equation of the DFE-PAP equalizer, given by the following formula [4,10]:

$$w(n) = w(n-1) + \mu[u^T(n)x(n)]^{-1}u(n)e(n) \quad (24)$$

Refer to [4] for additional details.

4.4 The Modified Recursive Non Quadratic Algorithm

The integration of the Modified Recursive Non-Quadratic (MRNQ) algorithm with the DFE structure results in the MRNQ-DFE equalizer [5]. The fundamental principle of the MRNQ-DFE is to adjust the filter coefficients toward their optimal values using a non-quadratic error function. This approach involves utilizing the incoming data at the equalizer input along with the instantaneous error. The objective of this process is to minimize the cost function and compute the filter gain for updating the filter coefficients [11].

The coefficient update equation for the MRNQ-DFE equalizer is given by [5]:

$$w(n) = w(n-1) + g(n)\gamma(n) \tag{25}$$

where:

The parameter $\gamma(n)$ represents a key modification introduced in this algorithm. It is incorporated into the update equation to enhance convergence behavior, and is defined as follows:

$$\gamma(n) = \frac{\varepsilon(n)}{|1+\varepsilon(n)|} \tag{26}$$

$\varepsilon(n)$ represents the preliminary estimation of filtering errors:

$$\varepsilon(n) = \frac{\tilde{a}(n-\Delta)}{(2j-1)} - w(n-1)^T x(n) \tag{27}$$

The term $g(n)$ represents the filter gain and is defined as follows:

$$g(n) = \frac{\lambda^{-1} B^{-1}(n-1)\boldsymbol{x}(n)}{\left[(2j-1)\tilde{a}(n-\Delta)^{2j-2}\right]^{-1} + \lambda^{-1}\boldsymbol{x}(n)^{\boldsymbol{T}} B^{-\mathbf{1}}(n-1)\boldsymbol{x}(n)} \tag{28}$$

$x(n)$ the channel output signal
$a(n-\Delta)$ the training sequence
$\tilde{a}(n-\Delta)$ the output of the decision block
L is the filter length
λ is the exponential weighting factor, $0 < \lambda < 1$
j is a positive integer

and:

$$\mathrm{B}(n) = \lambda \mathrm{B}(n-1) + \left[(2j-1)\tilde{a}(n-\Delta)^{2j-2}\right]x(n) \tag{29}$$

which is derived by calculating the gradient of $J(n)$ using the method of Lagrange multipliers:

$$\frac{\partial \mathrm{J}(n)}{\partial w(n)} = -2j\left[\sum_{i=1}^{n}[\lambda^{n-i}\tilde{a}(i-\Delta)^{2j-1}x(i)] - (2j-1)\sum_{i=1}^{n}[\lambda^{n-i}\tilde{a}(i-\Delta)^{2j-2}x(i)x(i)^{\boldsymbol{T}}]\, w(n)\right] \tag{30}$$

where this relation is simplified by splitting it into two terms:

$$\frac{\partial J(n)}{\partial \boldsymbol{w}(n)} = -2j\,[A(n) - \mathrm{B}(n)w(n)] \tag{31}$$

where
$$\begin{cases} A(n) = \sum_{i=1}^{n}[\lambda^{n-i}\,\tilde{a}(i-\Delta)^{2j-1}x(i)] \\ \mathrm{B}(n) = (2j-1)\sum_{i=1}^{n}[\lambda^{n-i}\tilde{a}(i-\Delta)^{2j-2}x(i)\,x(i)^T] \end{cases} \tag{3.27}$$

By setting $\frac{\partial J(n)}{\partial \boldsymbol{w}(n)} = 0_{L\times 1}$, we obtain:

$$w(n) = \mathrm{B}^{-1}(n)A(n) \tag{32}$$

For more details see [5].

5. Computational complexity

Table 1 presents a detailed comparison of the computational complexity (in terms of multiplications and divisions) of the four adaptive algorithms studied, as a function of the filter length L and projection order P. This analysis highlights the efficiency and practicality of each method for real-time applications.

Table 1. Computational complexity comparison.

Complexity per iteration for the NLMS-DFE:	3L+1
Complexity per iteration for the APA-DFE:	$LP^2+2LP+P^2+P$
Complexity per iteration for the DFE-PAP:	$2L+P^2+5P+4$
Complexity per iteration for the MRNQ-DFE :	$4L^2+3L$

The results reveal significant differences in computational load among the equalizers. Among the proposed techniques, DFE-PAP exhibits the lowest complexity after NLMS-DFE, since the projection order P in the DFE structure can be chosen much smaller than the filter length L, reducing the number of required operations.

6. Simulation results

In this section, we present a comparative analysis of the four equalizers: NLMS-DFE, APA-DFE, DFE-PAP, and MRNQ-DFE. This analysis is based on simulation results, with performance evaluated according to the following criteria:

- Constellation diagram
- Nyquist criterion
- Mean Squared Error (MSE)

The evaluation is carried out using two modulation schemes: 16-PSK and 16-QAM.

After conducting extensive experiments demonstrating the effectiveness of the proposed equalizers over a wide range of parameter values, including the step size µ, filter length L, predictor order P, and signal-to-noise ratio (SNR), we present the results obtained with the parameter set summarized in Table 2. These values were selected because they are common to all four equalizers and yield consistently good performance. This choice does not imply that the systems fail under other parameter settings; rather, it ensures a fair comparison and highlights representative configurations that provide satisfactory performance across all techniques.

Table 2. Parameters of the four Equalizers.

Algorithms	NLMS-DFE	APA-DFE	DFE-PAP	MRNQ-DFE
Parameters	SNR=40 dB L=64 μ=0,8	SNR=40dB L=64 P=8 μ=0,8	SNR=40 dB L=64 P=8 μ=0,8	SNR=40 dB L=64 $\lambda = 0.6$

6.1 Constellation criterion

In this section, we present the ideal, distorted, and equalized constellation diagrams obtained using the four equalizers: NLMS-DFE, APA-DFE, DFE-PAP, and MRNQ-DFE, as shown in Figures 3 and 4, for 16-PSK and 16-QAM modulation schemes, respectively.

As illustrated in Figures 3 and 4, the constellations obtained with all four equalizers exhibit a strong resemblance to the ideal reference for both 16-PSK and 16-QAM modulations. This result highlights the effectiveness of the proposed equalizers in significantly mitigating inter-symbol interference, thereby ensuring reliable signal reconstruction in digital communication systems.

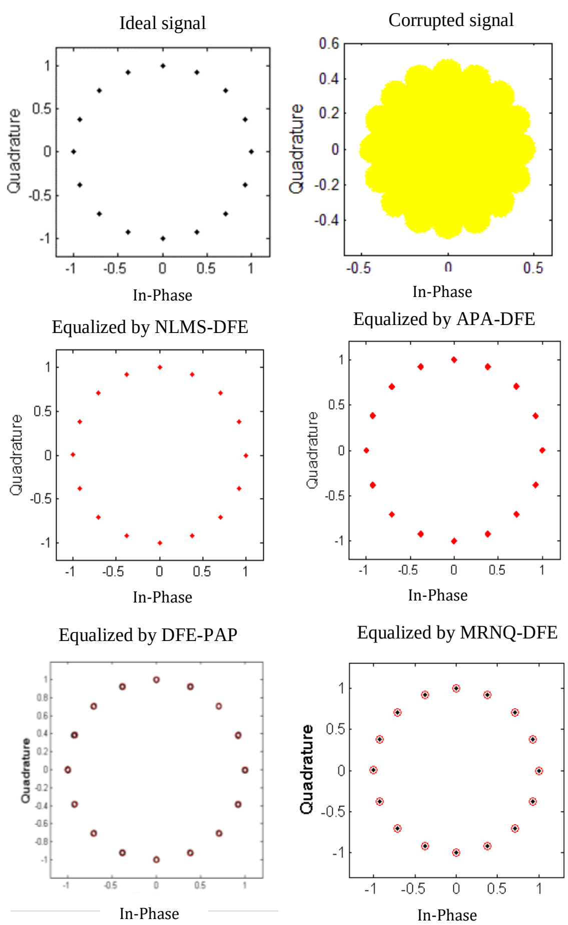

Figure 3. Constellation diagrams evaluation using the four equalizers under 16-PSK modulation.

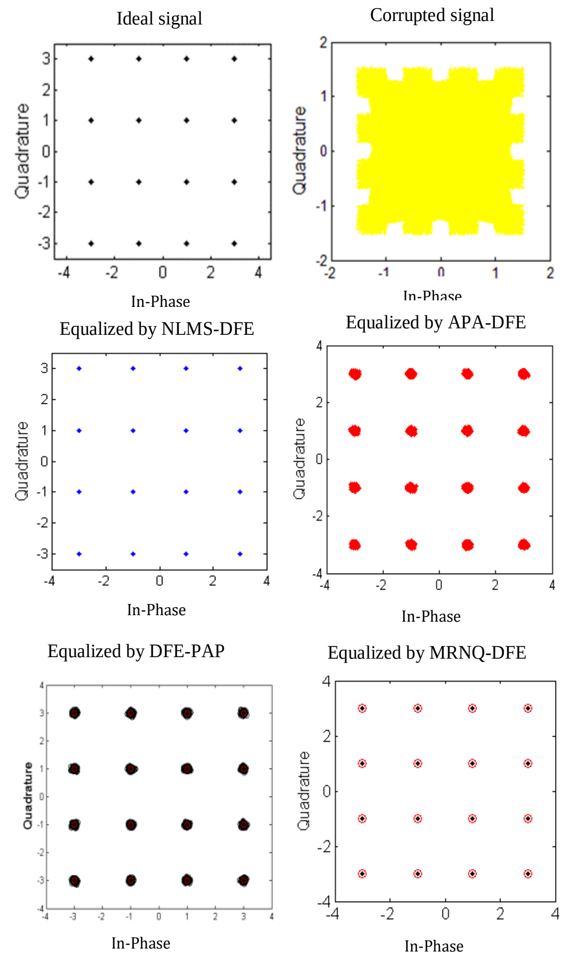

Figure 4. Constellation diagrams evaluation using the four equalizers under 16-QAM modulation.

6.2 Nyquist criterion

To assess the performance of all the proposed equalizers in channel equalization, the Nyquist criterion was employed. This criterion was evaluated by computing the convolution between the actual channel impulse response and the estimated one. The experiments were conducted using the same parameters as those presented in Table 2 for the NLMS-DFE, APA-DFE, DFE-PAP, and MRNQ-DFE equalizers. The Nyquist criterion was then calculated for each equalizer under two modulation schemes: 16-PSK and 16-QAM.

Figures 5 and 6 demonstrate the effectiveness of all four equalizers in compensating for the channel, resulting in an interference-free signal for both modulation schemes. This outcome makes the comparison between the techniques more challenging, as each successfully achieves the intended objective.

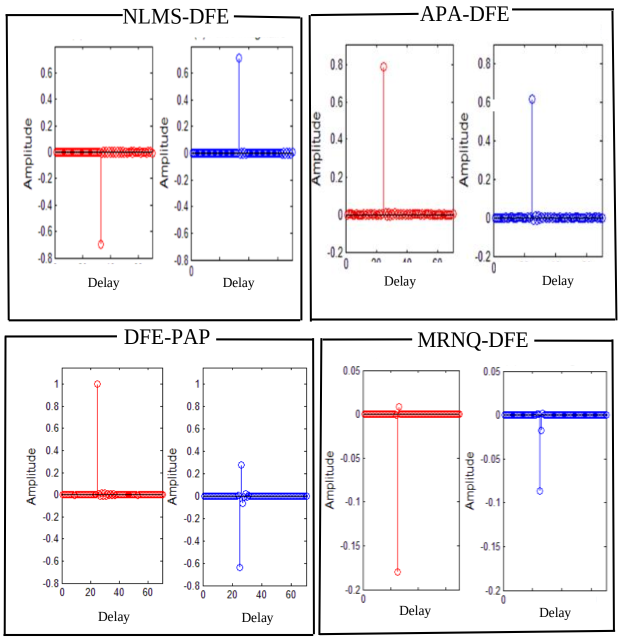

Figure 6. Nyquist criterion evaluation using the four equalizers under 16-QAM modulation. The red impulsion represents the real part, and the blue impulsion represents the imaginary part.

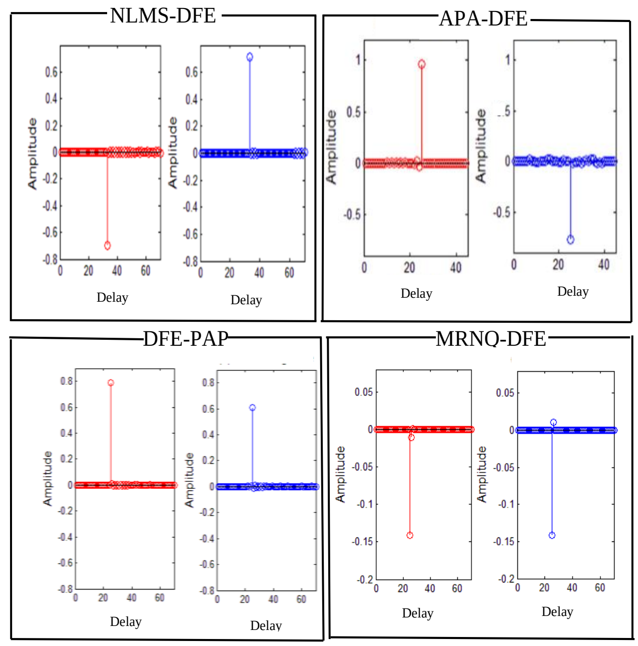

Figure 5. Nyquist criterion evaluation using the four equalizers under 16-PSK modulation. The red impulsion represents the real part, and the blue impulsion represents the imaginary part.

It is not possible to determine which of the four algorithms performs best based solely on the comparative results of the constellations and the Nyquist pulses shown in Figures 3, 4, 5 and 6.

However, all four equalizers effectively eliminate ISI in steady state and produce a well-equalized signal at the output. Therefore, the Mean Square Error (MSE) criterion is used to identify which algorithm offers the fastest convergence speed.

6.3 MSE Criterion

To rigorously assess the performance of each equalizer, we analyzed the convergence speed toward the minimum mean squared error (MSE) achieved by each, namely NLMS-DFE, APA-DFE, DFE-PAP, and MRNQ-DFE. This evaluation is illustrated in Figure 7 for 16-PSK modulation and in Figure 8 for 16-QAM modulation. The parameters used for each equalizer are listed in Table 2.

Figure 7 illustrates the convergence behavior of the four equalizers: NLMS-DFE, APA-DFE, DFE-PAP, and MRNQ-DFE, in terms of MSE under 16-PSK modulation. As observed, the MRNQ-DFE algorithm achieves the fastest convergence toward the minimum MSE, although it exhibits higher fluctuations in the steady state, indicating sensitivity to noise or signal variations. The DFE-PAP and APA-DFE algorithms also converge rapidly, reaching a steady-state level within a few iterations. In contrast, the NLMS-DFE converges more slowly and stabilizes at a substantially higher MSE level, reflecting its limited performance in complex modulation scenarios.

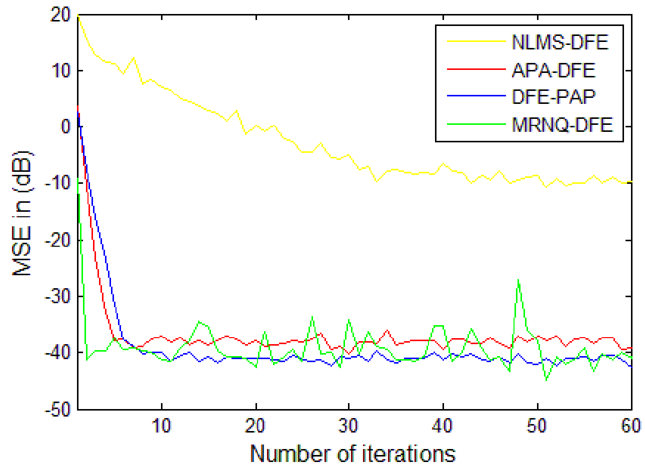

Figure 7. Mean Square Error analysis for the four equalizers using 16-PSK modulation.

Figure 8 displays the MSE convergence behavior of the four equalizers for 16-QAM modulation. The results clearly demonstrate that the MRNQ-DFE algorithm achieves the fastest convergence toward the minimum MSE, although its steady-state curve exhibits noticeable fluctuations due to the non-quadratic nature of its adaptation strategy. The DFE-PAP and APA-DFE algorithms also converge rapidly, with DFE-PAP providing particularly stable performance over the iterations. In contrast, NLMS-DFE converges more slowly and settles at a higher MSE level, reflecting its relatively limited performance under these conditions.

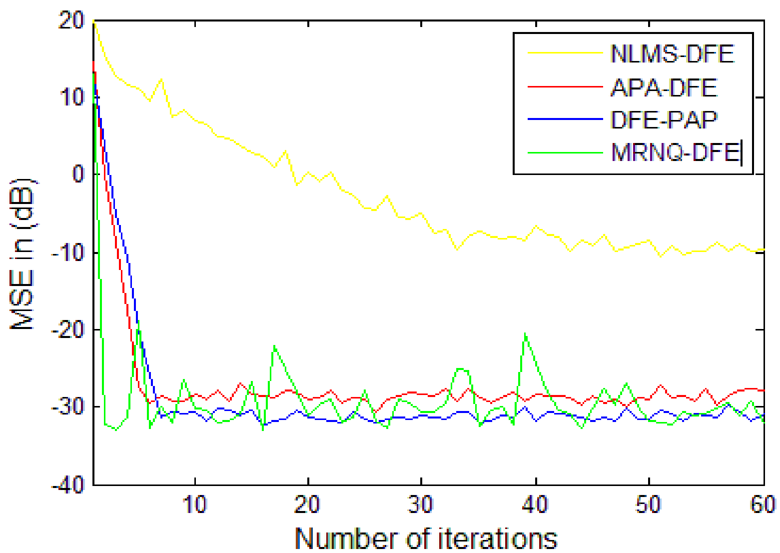

Figure 8. Mean Square Error analysis for the four equalizers using 16-QAM modulation.

These figures confirm the robustness of DFE-PAP and APA-DFE in terms of convergence stability and precision, while MRNQ-DFE achieves the fastest adaptation despite some fluctuations. NLMS-DFE remains the least effective in this scenario.

Table 3 presents a summary of the performance of the four equalizers based on the results discussed above.

Table 3. Summary of the performance of the four equalizers

Criterion	NLMS-DFE	APA-DFE	DFE-PAP	MRNQ-DFE
ISI mitigation	Good	Good	Good	Good
Convergence speed	Low	Medium	High	Very high
Minimum MSE	High	Low	Very Low	Very Low
comutational complexity	Low	High	Very low	Very high
Steady-state behavior	Slightly unstable	Stable	Stable	Slightly fluctuating

7. Conclusions

This paper introduces a decision feed-forward equalizer (DFE) model that integrates a training sequence along with four adaptive algorithms: the classical NLMS-DFE, the conventional APA-DFE, the novel DFE-PAP, and the new MRNQ-DFE. To evaluate the performance of these equalizers, we conducted a comparative analysis based on the constellation diagram, the Nyquist criterion, the mean squared error (MSE), and computational complexity.

The results confirm that all four algorithms effectively mitigate inter-symbol interference (ISI) and provide reliable signal equalization. Among them, DFE-PAP stands out as the most efficient, offering fast convergence, stable performance, and low computational complexity, making it highly suitable for practical implementation. APA-DFE also demonstrates strong performance, converging rapidly with good accuracy. MRNQ-DFE achieves extremely fast convergence, although it requires higher computational effort due to its non-quadratic adaptation strategy. In contrast, NLMS-DFE exhibits the lowest performance under the tested conditions.

Overall, the study highlights DFE-PAP as the best choice for balancing convergence speed, accuracy, and computational efficiency, while MRNQ-DFE offers ultra-fast adaptation at the cost of increased computational complexity

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